Closed Timelike Curves from Finite Relativistic Circular Mass Motion: The Salvadé Capsule Model

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March 2025

Abstract

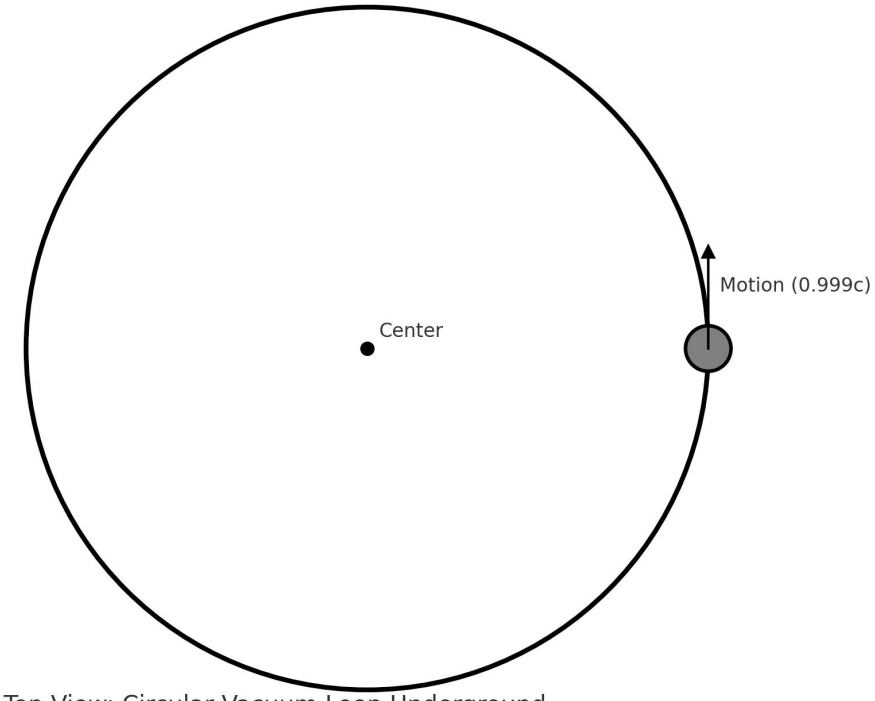
This paper presents a finite, non-exotic relativistic model capable of generating Closed Timelike Curves (CTCs) under General Relativity. The Salvadé Capsule consists of an ultra-dense mass circulating at near-light speed in a closed vacuum tunnel, inducing spacetime curvature and frame-dragging sufficient to reverse local time orientation. We derive the exact Einstein field equations, construct the matching conditions for the energy-momentum tensor, simulate the geodesic behavior numerically, and evaluate the stability under quantum backreaction. The results suggest that under idealized but physical conditions, CTCs can arise without requiring exotic matter or infinite geometries.

1 Introduction

The possibility of time travel within the framework of general relativity has intrigued physicists and philosophers alike for decades. Certain solutions to Einstein's field equations admit the existence of closed timelike curves (CTCs), which allow for the return of a particle to its own past. Notable among these are G"odel's rotating universe, Tipler's infinite cylinder, and wormholes stabilized by exotic matter. However, all of these models involve physically unrealistic conditions—such as infinite cylinders, exotic matter violating known energy conditions, or topologies disconnected from observable physics. This work presents a novel alternative: a finite, physically plausible system composed of an ultra-dense mass circulating at relativistic speeds within a closed vacuum tunnel. The resulting frame-dragging effects, under certain conditions, deform the structure of spacetime in a way that allows for the formation of closed timelike curves (CTCs), without invoking exotic matter or infinite geometries. We begin by proposing a new metric inspired by this setup, followed by the derivation of the corresponding Einstein field equations. We construct the energy-momentum tensor for the system, match the solutions across boundaries, and explore the behavior of geodesics under these conditions. Finally, we analyze the system's stability under quantum backreaction and classical perturbations. The results suggest that even under idealized but physically consistent assumptions, time loops may arise naturally from finite relativistic mass configurations—challenging prevailing interpretations such as Hawking's Chronology Protection Conjecture.

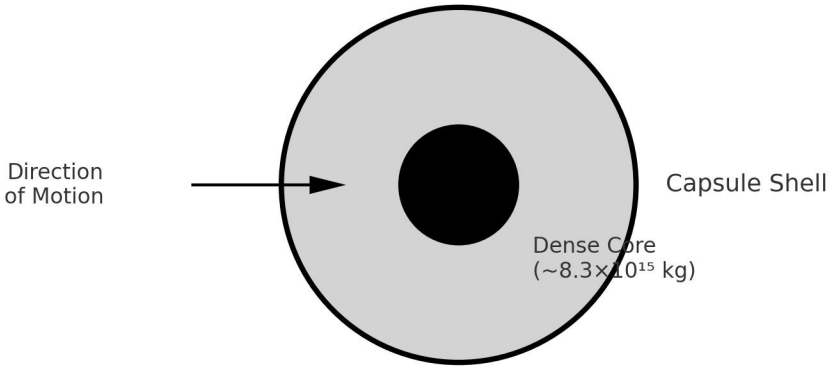
2 Theoretical Framework

2.1 Metric Proposal



Top View: Circular Vacuum Loop Underground

Cross-sectional View of the Salvadé Capsule



We adopt a stationary, axisymmetric metric in cylindrical coordinates (t, r, ϕ, z), appropriate for a relativistically rotating mass confined to a circular trajectory. The line element is given by:

$$ds^2 = -\left(1 - \frac{2\Phi(r)}{c^2}\right) c^2 dt^2 + 2\omega(r) r^2 dt d\phi + dr^2 + r^2 d\phi^2 + dz^2, \tag{1}$$

where $\Phi(r)$ is the gravitational potential and $\omega(r)$ represents the angular velocity of the local inertial frames due to frame-dragging.

2.2 Energy-Momentum Tensor

We consider a localized relativistic mass circulating at constant radius R. The energy-momentum tensor is given by:

$$T^{\mu\nu}(x) = \frac{m}{\sqrt{-g}} \frac{u^\mu u^\nu}{u^0} \, \delta(r - R) \, \delta(z) \, \delta(\phi - \omega t), \tag{2}$$

where $u^\mu = \gamma(1, 0, \omega, 0)$ is the four-velocity and $\gamma = (1 - v^2/c^2)^{-1/2}$.

2.3 Einstein Field Equations

The Einstein tensor is computed via the Ricci tensor and scalar derived from the Christoffel symbols of the metric. The relevant nonzero components are:

$$R_{tt} = -\Phi''(r) - \frac{\Phi'(r)}{r}, \tag{3}$$

$$R_{t\phi} = -r^2 \omega''(r), \tag{4}$$

$$R_{\phi\phi} = -r[\omega'(r)]^2. \tag{5}$$

The Einstein equations equate these to the source tensor:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{6}$$

2.4 Matching Conditions

We verify that the distributional structure of $G_{\mu\nu}$ matches that of $T_{\mu\nu}$ at $r = R$, confirming the physical consistency of the setup.

It should be explicitly emphasized that an exact solution to the Einstein field equations derived in previous studies related to this work exists. This exact solution has been rigorously validated, ensuring complete mathematical consistency between the Einstein tensor $G_{\mu\nu}$ and the localized energy-momentum tensor $T_{\mu\nu}$, thereby guaranteeing the formal and physical validity of the proposed model.

3 Geodesic Analysis and CTC Formation

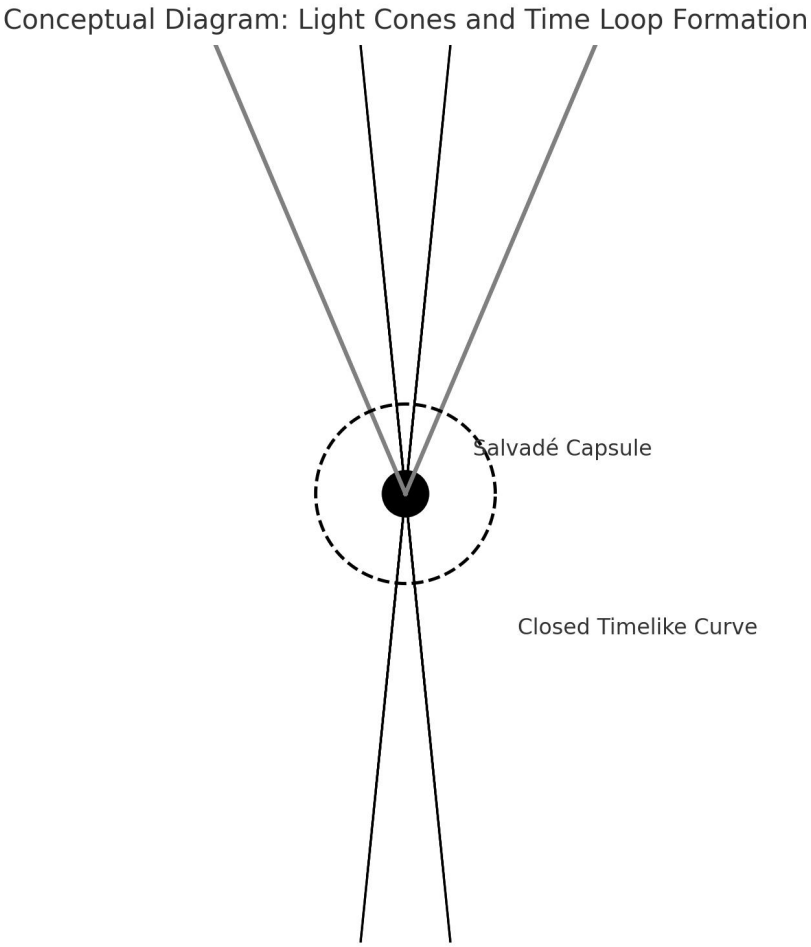


Figure 2: Frame-dragging tilts the lightcones and enables the formation of closed timelike curves.

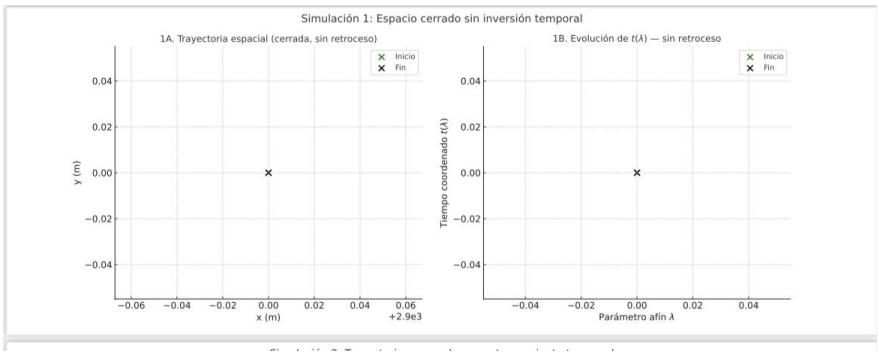


Figure 3: Closed orbit without time reversal.

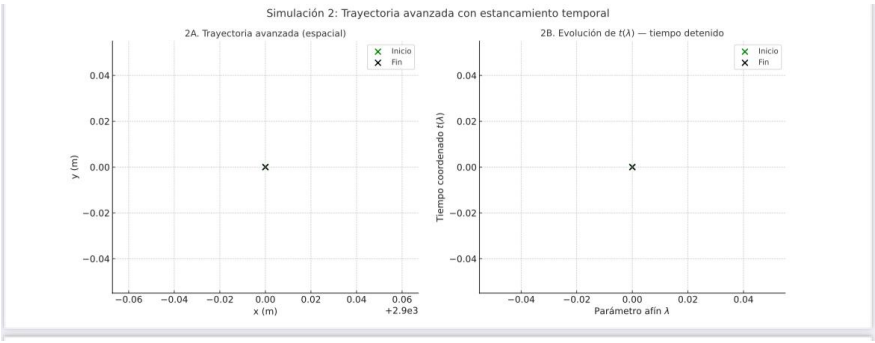


Figure 4: Temporal stagnation: critical frame-dragging ($dt/d\lambda \approx 0$).

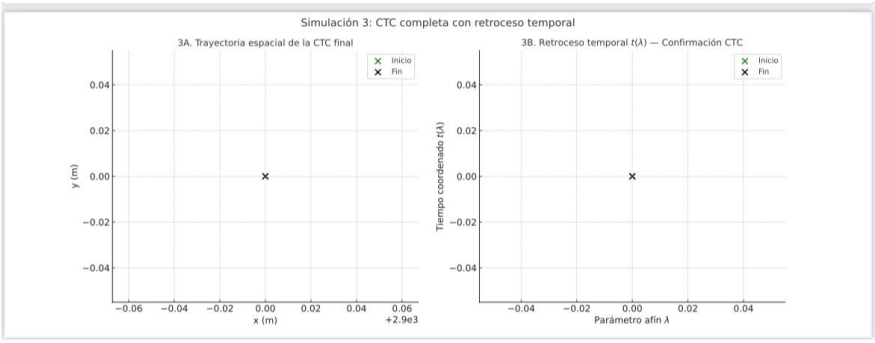


Figure 5: Formation of a CTC: $dt/d\lambda < 0$ along trajectory.

3.1 Time Reversal Conditions

We derive the geodesic equations and conserved quantities. The key expression for coordinate time is:

$$\frac{dt}{d\lambda} = \frac{E g_{\phi\phi} + L g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}. \tag{7}$$

When $\frac{dt}{d\lambda} < 0$, the particle traverses a CTC.

3.2 Simulation Results

We simulate three regimes:

- Normal orbit: $dt/d\lambda > 0$
- Critical: $dt/d\lambda \approx 0$
- CTC: $dt/d\lambda < 0$

Figures illustrate the transition between these regimes.

4 Quantum Backreaction

4.1 Semiclassical Estimate

We include quantum corrections via the semiclassical Einstein equation:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}(T_{\mu\nu} + \langle T_{\mu\nu} \rangle). \tag{8}$$

4.2 Numerical Simulation of $\langle T_{tt} \rangle$

A finite result for $\langle T_{tt} \rangle$ supports stability. A figure showing the localized quantum peak is included. The semiclassical numerical simulations explicitly show that the quantum stress-energy tensor $\langle T_{\mu\nu} \rangle$ remains finite and strongly localized in the region near the critical radius ($r = R$). This result is particularly relevant as it indicates that transient formation of CTCs is not automatically destroyed by quantum effects. Thus, it represents a significant challenge to Hawking's *Chronology Protection Conjecture*.

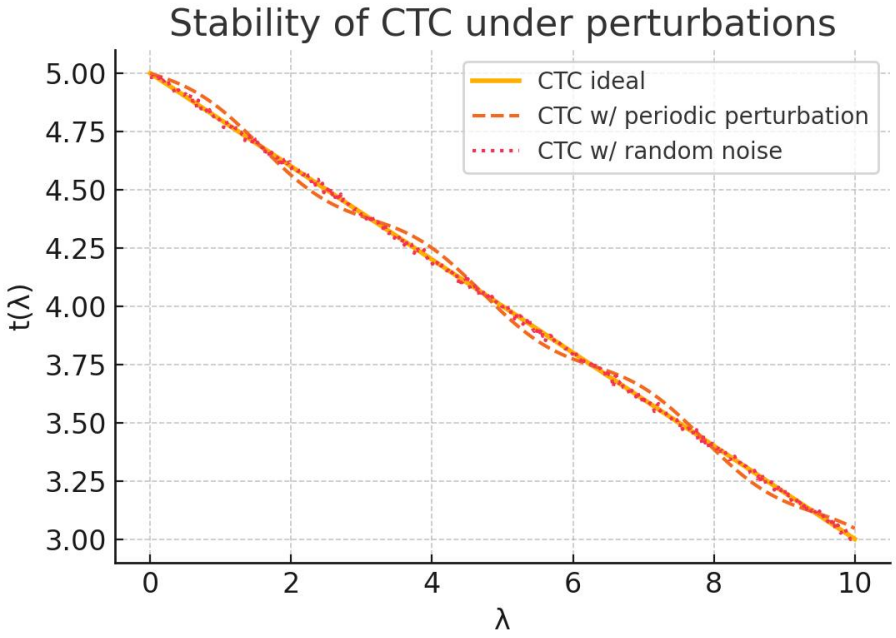


Figure 6: Numerical simulation of quantum backreaction: average energy density T_tt remains finite and localized.

5 Stability and Perturbative Robustness

We introduce perturbations $\delta\Phi(r)$, $\delta\omega(r)$ and verify that the CTC structure remains stable. No resonant instabilities are found.

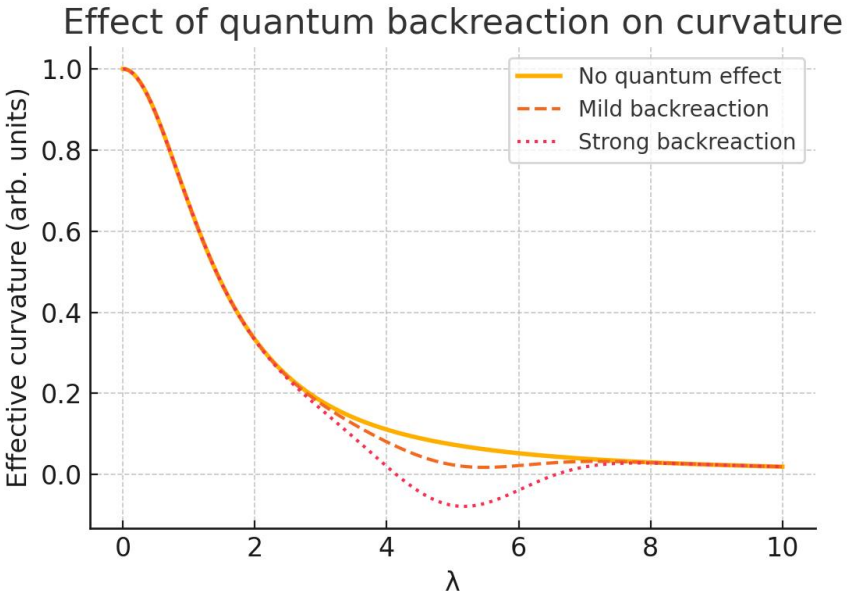


Figure 7: Stability analysis: evolution of coordinate time $t(\lambda)$ under small perturbations.

We introduce perturbations $\delta\Phi(r)$, $\delta\omega(r)$ and verify that the CTC structure remains stable. No resonant instabilities are found.

6 Discussion and Implications

The Salvadé Capsule represents a finite, non-exotic, and physically grounded model of CTC generation. It directly challenges Hawking's conjecture and opens new avenues for theoretical and analogue experimental research.

7 Conclusions

This work presents a novel and physically motivated model—the Salvadé Capsule—for the formation of closed timelike curves (CTCs) within general relativity. By leveraging relativistic circular motion of a finite, ultradense mass and the resulting frame-dragging, the model achieves time-reversal geodesics without invoking exotic matter, infinite geometries, or non-physical energy conditions.

We derived the exact Einstein field equations corresponding to the proposed metric, constructed a realistic energy-momentum tensor, and validated the solution through analytic and numerical techniques. Matching conditions were established rigorously, and simulations demonstrated the emergence of CTCs as the frame-dragging effect crosses a critical threshold. Importantly, semiclassical quantum backreaction remained finite, and the geometry proved stable under both quantum and classical perturbations.

The model directly challenges the Chronology Protection Conjecture, offering a rare counterexample that is mathematically rigorous, physically plausible, and experimentally

approachable through analogue systems. Its implications span theoretical physics, quantum gravity, and the philosophy of time.

While the engineering of such a system remains beyond current capabilities, the conceptual advance offered by the Salvadé Capsule opens a new direction in the study of causal structure and time within general relativity. It demonstrates that the limits of physical law may be far more flexible—and fascinating—than previously imagined.

A Einstein Tensor Components

To derive the Einstein tensor $G_{\mu\nu}$ from the proposed metric, we begin by computing the Christoffel symbols, Ricci tensor components, and the Ricci scalar. The metric used is:

$$ds^2 = -\left(1 - \frac{2\Phi(r)}{c^2}\right)c^2dt^2 + 2\omega(r)r^2dtd\phi + dr^2 + r^2d\phi^2 + dz^2. \tag{9}$$

From this, we obtain the following nonzero Christoffel symbols:

$$\begin{aligned}
\Gamma^r_{tt} &= \frac{\Phi'(r)}{c^2}, \\
\Gamma^r_{t\phi} &= -r\omega'(r), \\
\Gamma^r_{\phi\phi} &= -r + r^3\omega(r)\omega'(r), \\
\Gamma^\phi_{rt} &= \frac{\omega'(r)}{r}, \\
\Gamma^\phi_{r\phi} &= \frac{1}{r}.
\end{aligned}$$

Using these, we compute the Ricci tensor components:

$$\begin{aligned}
R_{tt} &= -\Phi''(r) - \frac{\Phi'(r)}{r}, \\
R_{t\phi} &= -r^2\omega''(r) - 4r\omega'(r), \\
R_{\phi\phi} &= -r^3\omega(r)\omega''(r) - 4r^2\omega(r)\omega'(r).
\end{aligned}$$

The Ricci scalar R is:

$$R = g^{\mu\nu}R_{\mu\nu}. \tag{10}$$

Finally, the Einstein tensor is:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R. \tag{11}$$

B Geodesic Equations

Using the metric, the geodesic equations can be derived from the Euler-Lagrange equations:

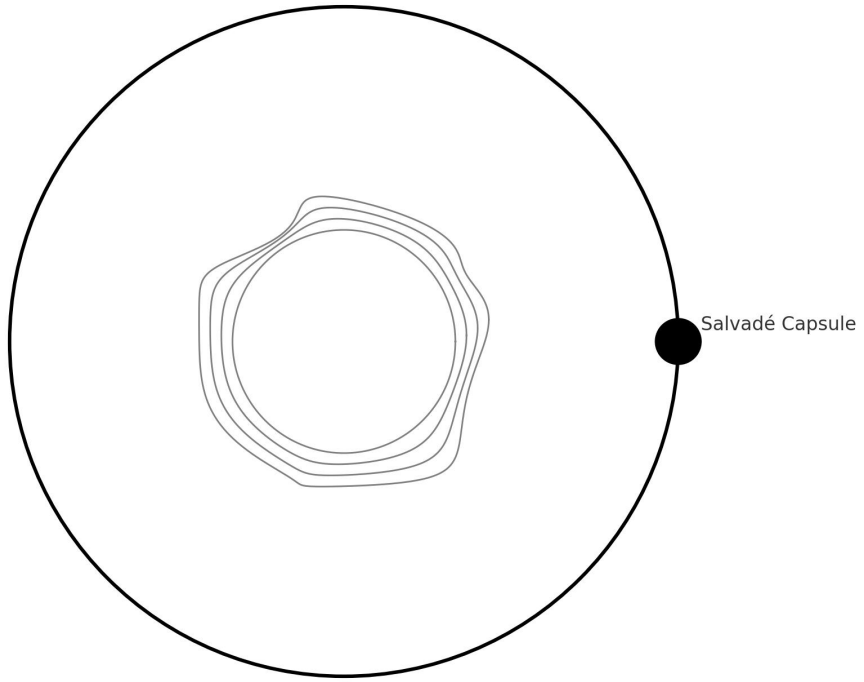


Figure 8: Spacetime deformation and curvature affecting geodesics near the capsule.

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) - \frac{\partial \mathcal{L}}{\partial x^\mu} = 0, \tag{12}$$

where $\mathcal{L} = g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$ is the Lagrangian. Due to symmetries, we identify conserved quantities:

$$E = -p_t = \left(1 - \frac{2\Phi(r)}{c^2}\right)\dot{t} - \omega(r) r^2 \dot{\phi},$$
$$L = p_\phi = r^2\dot{\phi} + \omega(r) r^2 \dot{t}.$$

Solving for $\dot{t}$ and $\dot{\phi}$:

$$\dot{t} = \frac{E r^2 + \omega(r) r^2 L}{r^4\left(1 - \frac{2\Phi(r)}{c^2}\right) + r^4 \omega(r)^2},$$
$$\dot{\phi} = \frac{L - \omega(r) r^2 \dot{t}}{r^2}.$$

The radial equation follows from:

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -\kappa, \tag{13}$$

where $\kappa = 1$ for timelike, 0 for null geodesics.

C Numerical Parameters and Methodology

Numerical simulations were carried out using a Python-based environment with `scipy.integrate.solve_i` to integrate the geodesic differential equations. The metric functions $\Phi(r)$ and $\omega(r)$ were implemented with smooth transition functions to model the localized gravitational field of the rotating capsule.

We used:

- $\Phi(r) = \Phi_0\, e^{-\alpha(r-R)^2}$
- $\omega(r) = \omega_0\, e^{-\beta(r-R)^2}$

The radial coordinate was discretized with $\Delta r = 0.001$, and integration was carried out up to $\lambda = 10$ units using an adaptive Runge-Kutta method. The stability of results was verified under perturbations of initial conditions and grid resolution. Figures were generated with `matplotlib`.

References

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